TOWARD A THEORY OF INFILTRATION IN A MEDIUM WITH DUAL POROSITY

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A study is made of the filling, with a fluid, of a model medium consisting of two interacting continua whose interpenetrating pore spaces are formed of pores that are roughly identical within the confines of the respective continua.

Beginning with the well-known study by Beckingham published eighty years ago, the movement of moisture in soils has traditionally been regarded as a diffusion process with a transport coefficient (diffusivity) dependent in the general case on moisture content (see [1, 2], for example). The most constructive feature of this approach is that it makes it possible to describe the empirically observed motion of a relatively steep moisture front at a finite velocity. The existence of wave solutions of this type for quasilinear parabolic equations was evidently first noted in [3]; the same subject was examined in [4] in regard to moisture waves generated by instantaneous sources.

Despite the fact that the moisture transport equations and the dependence of diffusivity on capillary potential and moisture content have been formulated using valid physical arguments regarding the water-retaining properties of soils, equilibrium conditions, and possible moisture transport mechanisms in soils [1], on the whole these equations are basically heuristic in character. Their solutions do not adequately reflect actual moisture transport processes in general or infiltration in particular. Thus, it has been known for more than two decades that the Clute equation and similar equations, if valid at all, are valid only for quasisteady processes [1, 5]. The theoretically determined velocity of mositure fronts [4] differs appreciably from the velocity seen in experiments [2]. In principle, the solutions of these equations cannot explain many features of moisture transport. This includes effects associated with capillary "suspension" of liquid, the true distribution of moisture in the moistened region, and the dependence of filtration velocity and the motion of the moisture front on the initial moisture content of the medium, the hydraulic head, etc. [2]. Thus, there is a general impression that the adopted method of describing moisture transport is inadequate and that it is necessary to develop new, physically more understandable representations and models free of the above shortcomings.

The pore space of actual soils and other porous media is an extremely complex structure from a topological viewpoint. It is usually modeled by representing the pores as constituting a system of interconnected capillaries or a system of interstices between close-packed particles. The capillaries and particles are assumed to be distributed somehow with respect to dimensions and form parameters. The simplest model which considers the difference between pores may be an aggregate of two systems of connected capillaries or particle interstices having their own characteristic structural scales. As a result, the porous medium is formally represented as a superposition of two coexisting continua having their own characteristic values of porosity and permeability and a capillary pressure which is independent of saturability. Such a model, similar to that used previously in the theory of filtration in cracked-porous media [6, 7], can be employed for an approximate description of granular soils. In the latter case, one of the continua is associated with flow through the interstices between granules, while the other is connected with flow in a system of contacting porous granules. The porosity of the first continuum is the volume fraction of interstices, while that of the second continuum is the volume fraction of pores in the granules. In a first approximation, the permeability of the first continuum is equal to the permeability of an analogous system with impermeable granules, while the permeability of the second continuum is proportional to the permeability of the material of the granules with a proportionality factor dependent on the actual contact area. This area can be evaluated form the model in [8].

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For both continua, we assume the validity of Darcy's law

$$\mathbf{v}_i = -(k_i/\mu)(\nabla p_i - \rho \mathbf{g}), \quad i = 1, 2, \tag{1}$$

while the volumetric flow between them, referred to a unit volume of the medium, is described by means of the quasisteady relation introduced in [6, 7]:

$$q = (\alpha/\mu)(p_1 - p_2). \tag{2}$$

The quantity α characterizes the structure of the porous medium; if we are dealing with a granular soil, then $\alpha = \beta k/a^2$, where k is the permeability of the material of the granules; a is their effective size; β is the numerical structural coefficient. The pressure undergoes a discontinuity equal to p_{di}. If the fluid is incompressible, then the following equations are valid in regions where both continua are filled with fluid

$$k_1 \Delta p_1 - \alpha (p_1 - p_2) = 0, \; k_2 \Delta p_2 - \alpha (p_2 - p_1) = 0, \tag{3}$$

these equations having been obtained in the usual manner form (1) and (2). In the regions where only one of the continua is filled, we have the usual Laplace equation for pressure in it.

To establish qualitative features of moisture transport within the framework of the proposed model, we will examine simple unidimensional problems concerning infiltration from a horizontal surface. The situation depicted in Fig. 1 corresponds to the downward flow of moisture from a free surface covered by a layer of water. The boundary conditions on the surface and the moisture fronts $z = h_i$ have the form

$$p_i = p_0, \; z = 0; \; p_i = -p_{ci}, \; z = h_i, \; i = 1, \; 2 \tag{4}$$

(positive p_{ci} correspond to the wetting fluid).

For the sake of definiteness, we will number the continua so that $h_2 > h_1$. Then, at $z = h_1$, we should impose the conditions of continuity of the function p_2 and its derivative with respect to z. If the fluid wets the material, then capillary forces assist its transport from the continuum with coarse pores to the continuum with fine pores. Thus, in this case the number i = 2 should probably be assigned to the last continuum.

The solution of the problem, following from (3) and (4) in the region $0 < z < h_1$ – where the fluid fills all of the pores – has the form

$$\begin{gathered} p_1 = -p_{c1} + (p_0 + p_{c1}) \left[1 - \frac{\operatorname{sh}(\lambda z)}{\operatorname{sh}(\lambda h_1)} \right] - R \left[\frac{z}{h_1} - \frac{\operatorname{sh}(\lambda z)}{\operatorname{sh}(\lambda h_1)} \right], \\ p_2 = p_0 + (p_0 + p_{c1}) \frac{k_1}{k_2} \frac{\operatorname{sh}(\lambda z)}{\operatorname{sh}(\lambda h_1)} - R \left[\frac{z}{h_1} + \frac{k_1}{k_2} \frac{\operatorname{sh}(\lambda z)}{\operatorname{sh}(\lambda h_1)} \right], \\ \lambda^2 = \alpha (k_1 + k_2)(k_1 k_2)^{-1}. \end{gathered} \tag{5}$$

The downward filtration velocities in the porous continua are obtained from (1) and (5):

$$\begin{gathered} v_1 = \frac{k_1}{\mu} \left\{ \rho g + (p_0 + p_{c1}) \lambda \frac{\operatorname{ch}(\lambda z)}{\operatorname{sh}(\lambda h_1)} + R \left[\frac{1}{h_1} - \lambda \frac{\operatorname{ch}(\lambda z)}{\operatorname{sh}(\lambda h_1)} \right] \right\}, \\ v_2 = \frac{k_2}{\mu} \left\{ \rho g - (p_0 + p_{c1}) \lambda \frac{k_1}{k_2} \frac{\operatorname{ch}(\lambda z)}{\operatorname{sh}(\lambda h_1)} + R \left[\frac{1}{h_1} + \lambda \frac{k_1}{k_2} \frac{\operatorname{ch}(\lambda z)}{\operatorname{sh}(\lambda h_1)} \right] \right\}. \end{gathered} \tag{6}$$

The filtration velocity, representing the volumetric flow of water into a porous medium per unit area of the free surface, is equal to

$$v_0 = v_1|_{z=0} + v_2|_{z=0}. \tag{7}$$

The velocities of the moisture fronts in the continua (i.e., in the different-size pores) are expressed as:

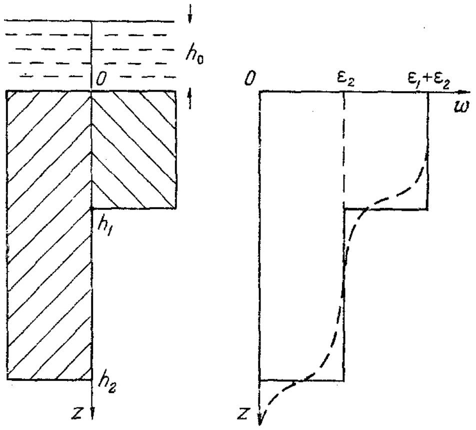

Fig. 1. Diagram of the penetration of moisture into pores represented by continua 1 and 2 (the moistening region is hatched) and diagram of the volumetric content of moisutre w in soil (dashed curve shows the typically-observed moisture distribution).

$$u_i = \frac{dh_i}{dt} = \frac{1}{\varepsilon_i} v_i\big|_{z=h_i}. \tag{8}$$

The relations obtained here describe the dynamics of infiltration with an accuracy R, which is a certain implicit function of time. To determine it, we need to solve the Laplace equation for p_2 in the region $h_1 < z < h_2$ — where only the second continuum is filled — and we need to use two continuity conditions at $z = h_1$ and the pressure discontinuity condition at $z = h_2$ from (4). As a result we obtain

$$R = \left\{1 + \frac{k_1}{k_2} + \left(\frac{1}{h_1} + \lambda\frac{k_1}{k_2}\operatorname{cth} x\right)(h_2 - h_1)\right\}^{-1} \times$$
$$\times \{p_0 + p_{c2} + (p_0 + p_{c1})(k_1/k_2)[1 + \lambda(h_2 - h_1)\operatorname{cth} x]\}, \quad x = \lambda h_1, \tag{9}$$

which finally determines fields (5) and (6). To find the functions $h_i(t)$, Eqs. (6), (8), and (9) readily yields a system of ordinary differential equations which we will write for dimensionaless coordinates of the fronts $x = \lambda h_1$ and $y = \lambda h_2$:

$$\frac{dx}{dt} = A[1 + H_1 \operatorname{cth} x + S(1 - x\operatorname{cth} x)],$$
$$\frac{dy}{dt} = B\left[1 - \frac{H_1}{\gamma}\operatorname{cth} x + S\left(1 + \frac{x}{\gamma}\operatorname{cth} x\right)\right],$$
$$S = \frac{\gamma H_2 + H_1[1 + (y - x)\operatorname{cth} x]}{1 + \gamma + (y - x)(\gamma + x\operatorname{cth} x)}, \tag{10}$$
$$A = \frac{\rho g \lambda k_1}{\varepsilon_1 \mu}, \quad B = \frac{\rho g \gamma k_2}{\varepsilon_2 \mu}, \quad \gamma = \frac{k_2}{k_1}, \quad H_i = \lambda\frac{p_0 + p_{ci}}{\rho g}, \quad i = 1, 2.$$

These equations completely describe the dynamics of the moisture fronts; in the general case, they must be solved by numerical means. Here, we will examine mainly two types of asymptotic motion of the fronts: when x and y increase without limit as $t \to \infty$, so that the difference $y - x$ remains finite; when only $y \to \infty$ as $t \to \infty$, while x approaches a finite limit.

In the first case, if we take (10) and ignore terms proportional to negative powers of x and y, we obtain:

$$\frac{dx}{dt} = A\left(1 - \frac{\gamma H_2 + H_1}{y - x}\right), \quad \frac{dy}{dt} = B\left(1 + \frac{\gamma H_2 + H_1}{\gamma(y - x)}\right)$$

and further

$$\frac{d(y-x)}{dt}=B-A+\left(A+\frac{B}{\gamma}\right)\frac{\gamma H_2-H_1}{y-x}.$$

It follows from this that one of the moisture fronts leads the other front by the dimensionless distance

$$\lim_{t\to\infty}(y-x)=(y-x)_*=(\gamma H_2+H_1)\frac{A+B/\gamma}{A-B}.$$

If $k_1H_1 + k_2H_2 > 0$, then the leading front is that in the continuum for which the quantity k_i/ε_i is lower (It is necessary that $A > B$). If the reverse inquality is satisfied, then the front in the continuum with the higher value of k_i/ε_i leads.

In dimensional variables, we have the following for the asymptotic distance between the fronts

$$(h_2-h_1)_*=\frac{\varepsilon_1+\varepsilon_2}{k_1\varepsilon_2-k_2\varepsilon_1}\,\frac{k_1(p_0+p_{c1})+k_2(p_0+p_{c2})}{\rho g}. \tag{11}$$

The asymptotic infiltration velocity and asymptotic velocities of the fronts are equal to:

$$u_*=u_{i*}=\frac{k_1+k_2}{\varepsilon_1+\varepsilon_2}\,\frac{\rho g}{\mu},\quad v_{0*}=(\varepsilon_1+\varepsilon_2)\,u_*. \tag{12}$$

The independence of these velocities on time means physically that after a certain amount of time has elapsed since the beginning of infiltration, the effect of capillary phenomena and the hydraulic head on the free surface becomes negligible compared to the effect of gravity.

Leaving only terms of the principal orders with respect to $y - x \approx y$ in (10), we obtain:

$$\frac{dx}{dt}=A\left[1+H_1\frac{(1+\gamma)\operatorname{cth}x}{\gamma+x\operatorname{cth}x}\right],$$

$$\frac{dy}{dt}=B\left\{1+\frac{1}{y}\left[H_2+\frac{H_1}{\gamma}\left(1-\frac{(1+\gamma)\operatorname{cth}x}{\gamma+x\operatorname{cth}x}\right)\right]\right\}.$$

It follows from the first equation that the regime being examined is possible at negative H_1, when $x(t)$ asymptotically approaches x_* as $t \to \infty$. Here, x_* is the root of the equation

$$\frac{\operatorname{cth}x_*}{\gamma+x_*\operatorname{cth}x_*}=\frac{1}{H},\quad H=|H_1|(1+\gamma). \tag{13}$$

At small γ, we have

$$x_*\approx H \quad \text{и} \quad h_{1*}\approx\frac{|p_0+p_{c1}|}{\rho g}. \tag{14}$$

Figure 2 shows the dependence of x_* on H and γ. Using (13), we then find

$$\left(\frac{dy}{dt}\right)_*=B\left(1+\frac{H_1+\gamma H_2-1}{\gamma y}\right).$$

It follows from this that at $H_1 + \gamma H_2 < 1$ the moisture front in the second continuum also asymptotically approaches the steady state

$$h_{2*}=\frac{y_*}{\lambda},\quad y_*=y^\circ=\frac{1-H_1-\gamma H_2}{\gamma}. \tag{15}$$

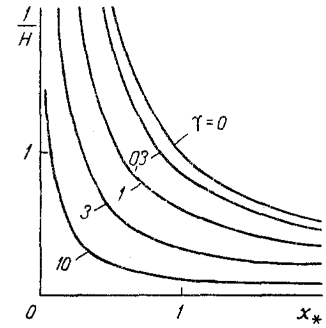

Fig. 2

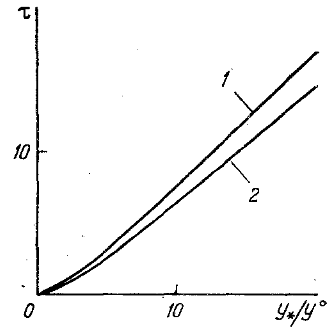

Fig. 3

Fig. 2. Dependence of x_* on H^{-1} with different γ.

Fig. 3. Dependence of y_*/y° on $\tau = (B/y^\circ)t$ in accordance with (16) and (17) (curves 1 and 2, respectively).

At $H_1 + \gamma H_2 > 1$, the front in the second continuum goes to infinity in accordance with the law

$$y_* - y^\circ \ln(1 + y_*/y^\circ) = Bt, \tag{16}$$

where y° was determined in (15). Examining the limiting expressions of this law at values of y_*/y° much greater and much less than unity, we find it reasonable to propose that y_* be determined from an approximate formula based on the superposition of these expressions (see Fig. 3):

$$y_* \approx (2y^\circ Bt)^{1/2} + Bt, \tag{17}$$

which in dimensional variables takes the form

$$h_{2*} \approx M\sqrt{t} + Nt, \quad N = \frac{\rho g k_2}{\varepsilon_2 \mu},$$
$$M = \left\{\frac{2}{\lambda \varepsilon_2 \mu}\left[k_1\left(\frac{\rho g}{\lambda} - p_0 - p_{c1}\right) - k_2(p_0 + p_{c2})\right]\right\}^{1/2}. \tag{18}$$

The infiltration velocity in the asymptotic regime being examined is calculated by means of (6) and (7):

$$v_0 = \frac{\rho g(k_1 + k_2)}{\mu}\left(1 + H_1 \frac{\operatorname{cth} x_*}{\gamma + x_* \operatorname{cth} x_*}\right) = \frac{\rho g k_2}{\mu}. \tag{19}$$

Equation (15) (and the corresponding regime) may only be of formal value, since the inequality $y \gg x$ needed to obtain the formula is not satisfied in this case. Thus, we should example only those situations when $H_1 + \gamma H_2 > 1$.

For a nonwetting liquid (negative p_{di}), the quantity H_1 may be less than zero at positive h_0 and $p_0 = \rho g h_0$ if $p_0 < |p_{d1}|$. This permits a unique physical interpretation. Together with the requirement $H_1 + \gamma H_2 > 1$, this leads to the condition

$$|p_{c1}| > p_0 > \frac{k_1}{k_1 + k_2}\left(\frac{\rho g}{\lambda} + |p_{c1}|\right) + \frac{k_2}{k_1 + k_2}|p_{c2}|.$$

It is clear from this that $p_0 > |p_{c2}|$ in any case, i.e., the moisture front for the nonwetting liquid is estabished in the continuum associated with the fine pores and it continues its movement in the coarse pores. In granular soils, this corresponds to flow in the interstices between granules.

For a wetting liquid, p_{ci} are positive, and H_1 can be negative only at negative p_0 corresponding not to a head but to a certain negative pressure on the free surface. In this case, the front stops moving in the coarse pores and moves in accordance with (16) in the fine pores.

The physical interpretation of processes with negative p_0 is not entirely clear. One could hope to formally connect the value of p_0 with the infiltration velocity – which can be assigned for a number of physical problems. However, in accordance with (19), infiltration velocity turns out to be independent of p_0. This means that the actual motion of moisture in the present case occurs while even the fine pores are unsaturated. The proposed model of a medium with dual porosity cannot be used to describe moisture transport under conditions of partial saturation, since it regards each continuum as being either saturated or unsaturated. (The characteristic moisture distribution is shown in Fig. 1.) However, although a more realistic model of a porous medium with a continuous pore-size distribution must be used to analyze the current situation, the result obtained here is an indication both of the general direction of development of the process and of the direction of necessary generalization of the proposed theory.

We emphasize that, as follows from the results, two mutually exclusive scenarios of development of the moistening process are possible at $H_1 < 0$. In the first of them, both fronts propagate without limit. In the second scenario, one of the fronts is stopped. The practical realization of one of these cases evidently depends on the initial conditions for h_i. In principle, the critical conditions corresponding to the shift in regimes can be found by solving Eq. (10) numerically. Physically, this means that given a sufficient depth of penetration of the moisture fronts, the weight of the liquid column in the pores will ensure its further downward movement.

Following the same reasoning, it is not hard to envision the propagation of the moisture upward from a submerged surface (as occurs in subsurface irrigation) and in the horizontal direction. In the first case, it is necessary to replace g by $-g$, while in the second case we take $g = 0$. In the case of upward propagation of mositure, at $t \to \infty$ the fronts approach steady states usually determined by hydrostatic equilibrium conditions with the involvement of forces associated with capillarity, gravity, and the pressure head. In the case of horizontal transport, either one or both fronts move away from the source in accordance with a parabolic law.

Let us briefly discuss the agreement between the results obtained here and experimental findings. Moisture-content diagrams such as the one shown in Fig. 1 by the dashed line have long been seen [9, 10] and are modeled fairly well within the framework of the present model by diagrams with moisture-content discontinuities. A semi-empirical formula identical in structure to that in (18) was also first proposed many years ago. In accordance with numerous experimental results such as those in [2], the limiting velocity of the front from (12) (or N from (18)) increases with an increase in initial moisture content. Meanwhile, infiltration velocity decreases in this case. The theory is consistent with the observations. In fact, the increase in the initial moisture content of the medium can be interpreted as a reduction in the effective porosity ε_2 and, thus, in k_2. For example, for a wetting liquid in granular soil, $k_1 >> k_2$ and $\varepsilon_1 \sim \varepsilon_2$, i.e., u_* from (12) actually increases and v_{0*} decreases with an increase in the initial moisture content of the granules. Also, in agreement with observations [21] is the dependence of the "sorptivity" M from (18) (to use the terminology in [2]) on the hydraulic head and capillary forces.

NOTATION

A, B, H_i, parameters introduced in (10); g, acceleration due to gravity; h_i, coordinates of the moisture front; k_i, permeability coefficients; M, N, parameters in (18); p_i, pressure; p_{ci}, capillary pressure; p_0, pressure at the surface-source; q, volumetric flow of fluid from the first continuum into the second; R, S, quantities determined in (9) and (10); t, time; u_i, velocities of the fronts; v_i, filtration velocities; v_0, infiltration velocity; x, y, dimensionless coordinates of the fronts; z, vertical coordinate; α, exchange coefficient introduced in (2); ε_i, porosity; λ, parameter determined in (5); μ, viscosity; ρ, density. The subscripts 1 and 2 denote quantities pertaining to the different continua; * denotes asymptotic values of the variables.

LITERATURE CITED

1. S. V. Nerpin and A. F. Chudnovskii, Physics of Soil [in Russian], Moscow (1967).
2. J. R. Phillip, Isothermal Movement of Moisture in the Aeration Zone [Russian translation], Leningrad (1972), pp. 6-81.
3. Ya. B. Zel'dovich and A. S. Kompaneets, In Observance of the Seventieth Birthday of A. F. Ioffe, Moscow (1950), pp. 61-71.

4. Yu. A. Buevich and U. M. Mambetov, Inzh.-Fiz. Zh., 54, No. 6, 982-988 (1988).
5. M. Hallaire, L'eau et la Production Vegetale, Inst. Nat. Rech. Agronomique (1964).
6. G. I. Barenblatt and Yu. P. Zheltov, Dokl. Akad. Nauk SSSR, 132, No. 3, 545-548 (1960).
7. Yu. A. Buevich, Inzh.-Fiz. Zh., 46, No. 4, 593-600 (1984).
8. A. A. Belyaev, A. Yu. Zubarev, E. S. Kats, and V. M. Kiseev, Inzh.-Fiz. Zh., 55, No. 1, 122-130 (1988).
9. E. A. Colman and G. B. Bodman, Soil. Sci. Soc. Am. Proc., 9, No. 1, 3-11 (1945).
10. J. Crank and M. E. Henry, Trans. Faraday Soc., 45, 636-642 and 1119-1128 (1949).

INVESTIGATION OF DYNAMIC CHARACTERISTICS OF A GLOW DISCHARGE IN GAS FLOWS

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Stability of the gas-discharge-supply circuit system relative to current fluctuations is examined. The estimates are in agreement with investigations of a glow discharge in an air flow.

1. The stability of stationary flow discharge (GD) combustion must be assured for technological applications since current and voltage fluctuations result in reduction in the efficiency of the GD installation [1]. The fluctuations are due to voltage oscillations in the supply source, in parasitic capacitances and inductances of the loop, and also in the development of plasma instabilities in the discharge itself [1-3]. Hence, the stability of GD combustion or the damping of current fluctuations are governed by the whole discharge-supply loop system.

To produce optimally efficient and stable gas-discharge installations the dependence of the nature of the fluctuation on the GD maintenance circuit must be established theoretically and from experiments.

The simplest loop contains an emf source ε and a ballast resistance R_b connected in series with the GD (Fig. 1). A "static" current—voltage characteristic (CVC) obtained experimentally by varying ε and recording the discharge voltage U and current i is used in the stability analysis. It follows from a classical examination [4-7] that the GD should combust stably if

$$R_b > |r|, \tag{1}$$

$$L > R_b |r| C, \tag{2}$$

where L is the inductance connected in series with the GD, C is the capacitance in parallel with the GD, r is the GD differential resistance (DR) equal to the slope of the tangent to the CVC at the working point under consideration $r = dU/di$.

The necessary condition (1) denotes that as the CVC ($r > 0$) grows the GD is always stable and the oscillations in the GD-supply loop system are quenched while for a dropping CVC ($r < 0$) the load line corresponding to the equation $\varepsilon = IR_b + U$ should pass more steeply than the tangent to the CVC at the working point. Satisfaction of conditions (1) and (2) is sufficient for damping the fluctuations.

Condition (1) and (2) correctly describe the excitation and quenching of oscillation within the framework of the circuit taken, however, their application in this form to estimate the specific gas discharge system is made difficult since dynamic fluctuations always

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